

Students' Addition Strategies

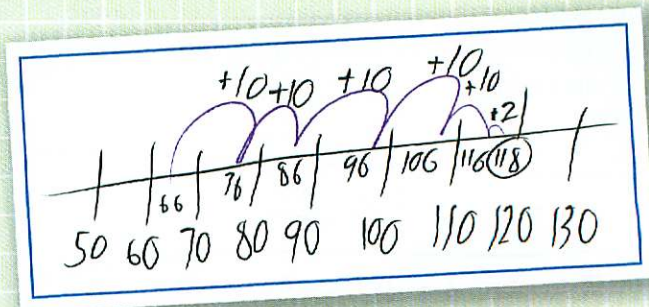
As students work on the activities in Investigation 4, you will see a wide range of understanding of the operation of addition. By the end of Grade 2, all students should be fluent with at least one strategy for solving 2-digit addition problems and notating their work. In Grade 2, fluency means that they have a strategy that they understand, that is efficient and accurate, and that works to solve addition problems with totals to just over 100. At this point, efficiency means that students are not counting on by ones. Students will vary in terms of flexibility; some are quite flexible, choosing a strategy based on the numbers in the problem, and others use one strategy exclusively.

Students' strategies for addition fall into three basic categories. Note that at this point in the year, it is assumed that students understand that addition is commutative and, therefore, that they can solve the following problem as $66 + 52$ or $52 + 66$:

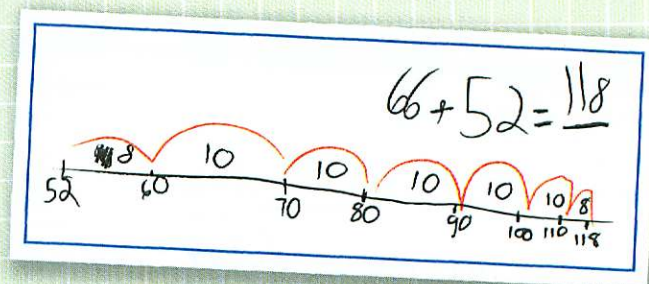
Franco had 66 car stickers. Jake gave him 52 car stickers. How many car stickers does Franco have now?

Keeping One Number Whole

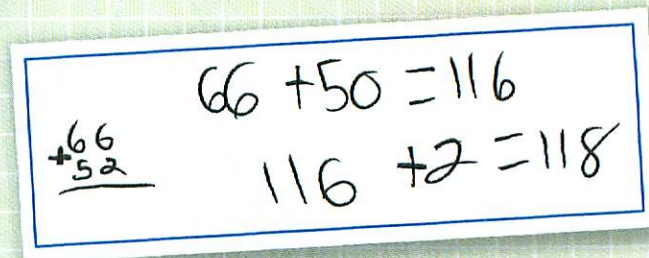
At the end of Grade 2, many students keep one number whole and add the other one *in parts*. Fluency with this strategy means that students add on the other number in parts; that is, they do not count on by ones. Some students add on by place, adding the tens first and then the ones. They may add each ten in the number, as Chen and Carla did, or all of the tens at once, as Darren did. Others are more comfortable adding on to a number that is a multiple of 10, as Simon did. They add on enough to the other number to get to a multiple of 10 and then proceed.



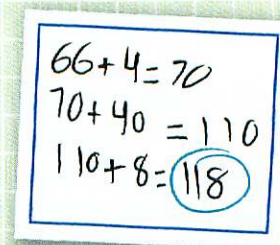
Chen's Work



Carla's Work



Darren's Work



Simon's Work

Students should be working to add the largest parts possible while still making sense of the problem and the numbers. For example, students who are adding each individual group of 10 should be working to add multiples of 10, such as 50 or 60.

Adding Tens and Ones

At the end of Grade 2, many students add by place to combine 2-digit numbers. Whether they use a place-value model (e.g., the sticker notation used by Holly) or equations, these students break both numbers into tens and ones. Most combine the tens first, a strategy that provides useful information about the approximate size of the total. Their second step varies. Some, like Esteban, combine the ones ($6 + 2$) and then the subtotals ($110 + 8$), and others, like Henry, add each group of ones, one by one, onto the total ($110 + 2 = 112$, $112 + 6 = 118$).

Holly's Work

Esteban's Work

Henry's Work

Some students may use a place-value strategy but add the ones first and then the tens. There are several ways that students notate this strategy, including the U.S. standard algorithm for addition.

Use vertical notation to model adding tens and ones, but we do suggest that, for demonstration purposes, teachers record what was added (e.g., $60 + 50 =$) to help students make sense of both the strategy and the notation.

Some students use vertical notation to record. As you model their strategies, emphasize that you are doing this for demonstration purposes. Students who use vertical notation do not need to write this extra information when they record.

The U.S. “carrying” algorithm, which some second grade students may be familiar with, is also an example of adding by place. Rather than beginning with the largest place, as students often do naturally, this algorithm begins with the smallest place. It includes a shorthand way of notating the value of numbers as the digits in each place are added. For many second graders, the compressed notation of this algorithm can obscure both the place value of the numbers and the meaning of each step of the procedure. This can lead to a more rote approach to solving addition problems, while students are still solidifying their understanding of the base-ten number system and the operation of adding in Grade 2—steps in students’ development of computational fluency that take time and practice.

After students have developed good, efficient algorithms that they understand and can carry out easily, such as adding by place, some may also become fluent in the traditional or standard algorithm. Others will continue to use adding by place or adding on in parts fluently, which will also serve them well for their computation needs now and as adults. The U.S. “carrying” algorithm is not addressed directly in Grade 2, although some students may be able to use it with understanding. Note that the vertical notation of adding by place value shown on the previous page, in which the ones are added first, is closely related to the steps in the standard algorithm but makes these steps more transparent. When students use the standard algorithm, demonstrate this form of notation and help students compare the two. Students who use the standard algorithm should also learn other strategies that demonstrate their flexibility with and understanding of addition. The U.S. algorithm is included in a study of strategies for addition in Grade 4.

Changing the Numbers to Make an Equivalent Problem

At the end of Grade 2, a few students may “take” an amount from one of the addends and “give” it to the other addend, creating an equivalent problem that is easier to solve. For example, Jacy takes six from the 66 and gives it to the 52.

$$66 + 52 = 60 + 58 = 118$$

Jacy's Work

Similarly, Nadia takes 2 from the 52 and gives it to the 66, and Lonzell takes 4 from the 52 to turn 66 into 70.

$$\cancel{66} + \cancel{52} = 68 + 50 = 118$$

Nadia's Work

$$\cancel{66} + \cancel{52} = 70 + 48 = 118$$

Lonzell's Work

Depending on how this strategy is conceptualized and carried out, it can also be seen as a variation of keeping one number whole and adding the other on in parts.

$$\begin{aligned}66 + 52 \\66 + 4 = 70 \\70 + 48 = 118\end{aligned}$$

In Grades 3 and 4, students will study this strategy and others in more depth.

Other Strategies

There are a few other strategies to watch for. For example, depending on the numbers in a particular problem, some students change one or both of the numbers, or work with nearby friendly or landmark numbers, and then compensate for the changes. For example, consider these students' work with $66 + 52$.

$$\begin{aligned}66 + 52 \\70 + 52 = 122 \\I \text{ added } 4 \text{ too many, so} \\ \text{subtract } 4 \\122 - 4 = 118\end{aligned}$$

Juanita's Work

To some, Juanita's strategy seems similar to changing the numbers to create an equivalent problem, and Nate's looks like a variation of adding by place. It is not important that everyone agree on categorizing student strategies; rather it is important that the strategy be accurate, efficient, and understandable to the student.

Flexibility

At the end of Grade 2, some students can use only one of the above strategies fluently. Others are comfortable enough with several strategies to choose a method based on the structure of the problem or the numbers in it. For example, you may see a student who adds on one number in parts when combining a large number and a small number but who otherwise adds by place, or you may see a student who always adds by place unless the problem involves numbers close to landmark numbers, such as 25 or 50. Developing fluency with all these strategies is a focus of the Grade 3 work on addition.

Notation

All students should be able to write an equation (horizontally or vertically) that represents a given problem situation and interpret and solve a problem presented in either horizontal or vertical form. However, second graders often use a variety of notational systems—including numbers, equations, the number line, and stickers as a model for representing the place value of our base-ten number system—to solve problems and record their work. See **Teacher Note:** Notating Addition Strategies, page 176 for more information.

$$66 + 52 = 118$$

If it was $50 + 50$ that would = 100 but there was an extra 10 from the 66 and that would equal 110 and $6 + 2$ equals 8 and $8 + 10 = 18$ so add a hundred and you get 118.

Nate's Work